

KALMAN FILTER APPLICATIONS

10.1: Examples of Kalman filters

- To wrap up the course, we look at several of the applications introduced in notes chapter 1, but in more detail.
- My students and I have been directly involved with these examples.

Tracking marker dots on actors

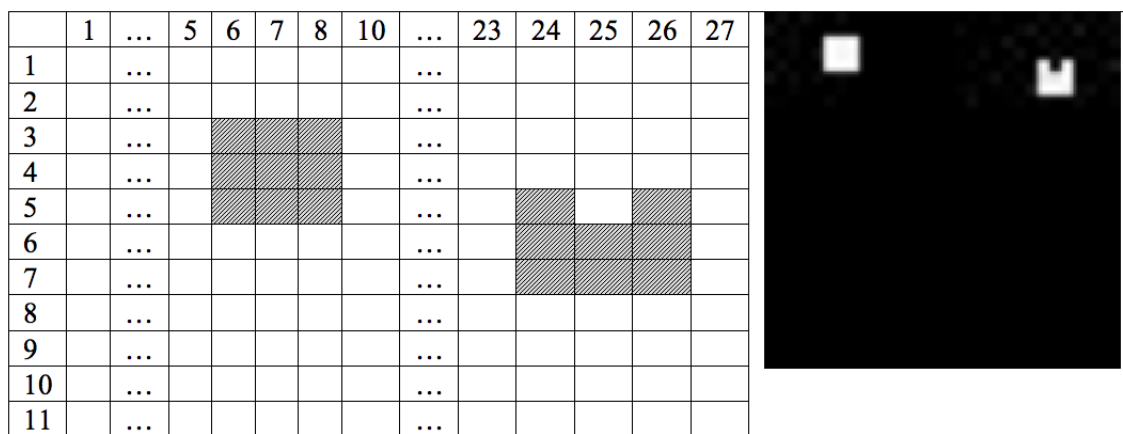
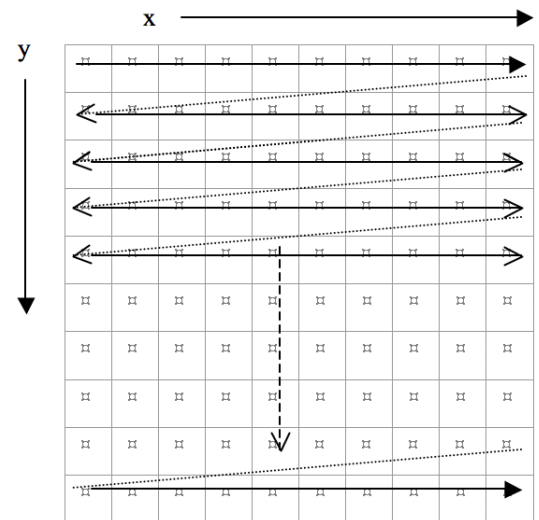
- State: x , y position and velocity of dots in frame.¹
 - Observation: x , y positions of dots in frame (unlabeled).
 - Issues: Data association, tracking when dots are obscured.
- Images containing actors with reflective marker dots arrive for processing at 30 frames per second.



- Dennis' first challenge was detecting targets in a 2D camera field in an efficient way.

¹ From, James Dennis Musick, *Target tracking a non-linear target path using a Kalman predictive algorithm*, MSEE thesis, University of Colorado at Colorado Springs, 2005.

- The standard NTSC scan order for when pixels arrive is shown to the right.
- Dennis created an efficient centroid calculation algorithm that worked in real time as the pixels arrived (in scan order), and can handle up to a pre-specified maximum number of simultaneous targets.
- The following is an example illustrating some of the issues



Two Target - Algorithm Centroid Output is (7.0, 4.0), (25.0, 6.125)

- After scanning row 5, there are three centroid candidates; after scanning row 6, two are joined, to leave only two candidates.
- The next issue was the target dynamic model to use.
 - Dennis tried both NCV and NCA models;
 - Sensor noise was determined to be on the order of 1/2 pixel;
 - $\Sigma_{\tilde{w}}$ was selected by evaluating the statistics of accelerations and jerks in a database of typical motion capture scenarios.
- The next issue was how to associate centroid position measurements to individual target tracks.

- Dennis used a maximum-likelihood association method. That is, for every centroid-target pair, he calculated

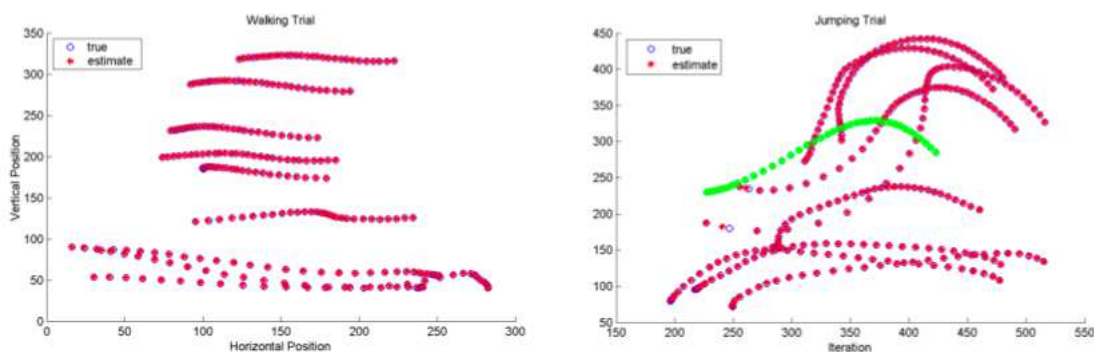
$$\text{likelihood } L = \frac{1}{(2\pi)^{n/2} |\Sigma_{\tilde{x}}|^{1/2}} \exp\left(-\frac{1}{2}(x - \bar{x})^T \Sigma_{\tilde{x}}^{-1}(x - \bar{x})\right),$$

where x is the centroid measurement, and \bar{x} is the target's present state estimate, and $\Sigma_{\tilde{x}}$ is the target's present covariance estimate.

- He then formed a table of likelihood values

Measured targets	Estimated targets				
	1	2	3	4	5
1	$L_{1,1}$	$L_{1,2}$	$L_{1,3}$	$L_{1,4}$	$L_{1,5}$
2	$L_{2,1}$	$L_{2,2}$	$L_{2,3}$	$L_{2,4}$	$L_{2,5}$
3	$L_{3,1}$	$L_{3,2}$	$L_{3,3}$	$L_{3,4}$	$L_{3,5}$
4	$L_{4,1}$	$L_{4,2}$	$L_{4,3}$	$L_{4,4}$	$L_{4,5}$

- He found the table maximum value, and made that association, and set all entries in that row and column to zero; he repeated until all measurements were accounted for.
 - Occluded targets (missing measurements) were handled by skipping measurement updates for those target tracks.
- Dennis found that the NCV model worked best for this application, and the results were outstanding.

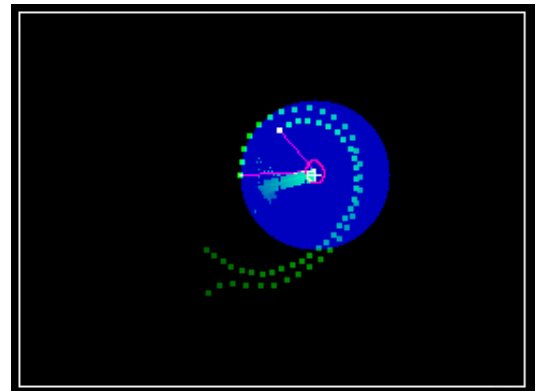


- Ongoing challenges in multi-target tracking:

- Efficiency of the data association process in particular, and of multi-target tracking in general.
- For example, there are an estimated 80,000,000 objects 1cm across or larger orbiting earth (large enough to disable a satellite).
- We presently track about 18,000 of the largest ones. Orbit estimation, collision prediction are hot topics, but very difficult too.
- Furthermore, beyond assessing where an object is, being able to say what it is doing and what that means are two very important questions to answer.

Localizing bad guys (or, search and rescue)

- State: x, y position and velocity^{2,3}
 - Observation: Direction (angle) from UAV to target.
 - Issues: Nonlinear relationship between measurements and position; measurements arriving to KF out-of-sequence.
- Multiple UAVs search a pre-defined geographic region for targets of interest.
- Heterogeneous sensors are used: passive radar, camera, IR.
- All sensors measure only angle to target (not x, y position, nor range to target).



² Plett, G., DeLima, P., and Pack, D.J., "Target Localization using Multiple UAVs with Sensor Fusion via Sigma-Point Kalman Filtering," in CD-ROM Proc. AIAA Convergence 2007, Rohnert Park, CA (May 2007), 15 pages.

³ Plett, G., Zarzhitsky, D., and Pack, D.J., "Out-of-Order Sigma-Point Kalman Filtering for Target Localization using Cooperating Unmanned Aerial Vehicles," Hirsch, M.J., Pardalos, P.M., Murphey, R., and Grundel, D. (eds), Advances in Cooperative Control and Optimization, Springer (Lecture Notes in Control and Information Sciences), 2007, [ISBN: 978-3-540-74354-5].

- For the targets, a modified NCV model was used

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{\eta}(t) \\ \ddot{\xi}(t) \\ \ddot{\eta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \\ \dot{\xi}(t) \\ \dot{\eta}(t) \end{bmatrix} + w(t)$$

$$z(t) = \text{atan2}(\eta_{\text{uav}}(t) - \eta_{\text{target}}(t), \xi_{\text{uav}}(t) - \xi_{\text{target}}(t)) + v(t).$$

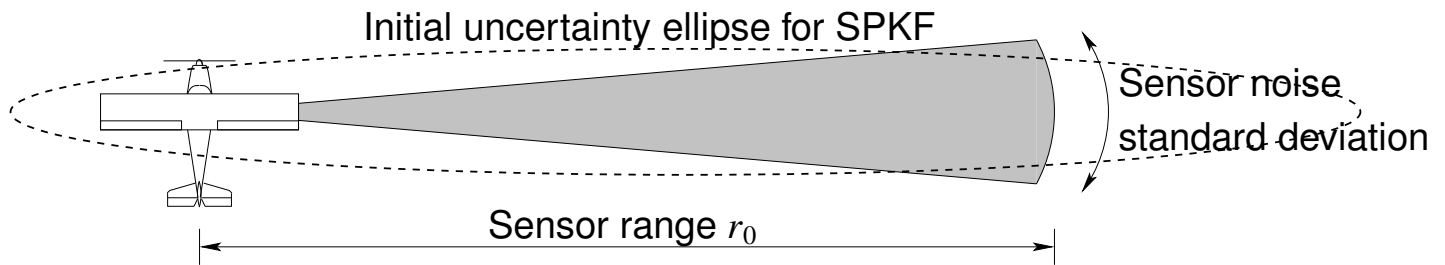
- Note (1) that the output equation is nonlinear, and (2) that a baseline continuous-time model is used since measurements are not necessarily aligned with a pre-defined sample rate.
- SPKF handles nonlinear output equation, but still needed to be very careful with modulo- 2π issues in measurements (a gigantic pain).
- The state equation, evaluated over a non-constant time interval, is

$$\begin{bmatrix} \xi(t + t_0) \\ \eta(t + t_0) \\ \dot{\xi}(t + t_0) \\ \dot{\eta}(t + t_0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_0 & 0 \\ 0 & 1 & 0 & t_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \\ \dot{\xi}(t) \\ \dot{\eta}(t) \end{bmatrix} + w_{t_0}.$$

- Process noise integrated over a non-constant time interval is incorporated as

$$\Sigma_{\tilde{w}_{t_0}} = \int_0^{t_0} e^{A(t_0-\tau)} \Sigma_{\tilde{w}} e^{A^T(t_0-\tau)} d\tau = \begin{bmatrix} \frac{t_0^3 \Sigma_{\tilde{w}}}{3} & 0 & \frac{t_0^2 \Sigma_{\tilde{w}}}{2} & 0 \\ 0 & \frac{t_0^3 \Sigma_{\tilde{w}}}{3} & 0 & \frac{t_0^2 \Sigma_{\tilde{w}}}{2} \\ \frac{t_0^2 \Sigma_{\tilde{w}}}{2} & 0 & t_0 \Sigma_{\tilde{w}} & 0 \\ 0 & \frac{t_0^2 \Sigma_{\tilde{w}}}{2} & 0 & t_0 \Sigma_{\tilde{w}} \end{bmatrix}.$$

- Initializing the target state using a single measurement of arrival angle was an issue



- We assume a uniform distribution on $R \sim \mathcal{U}(0, r_0)$, where r_0 is the sensor range.
- We model the sensor reading $\hat{\Theta} = \Theta + \Theta_{\text{noise}}$ where Θ_{noise} is a Gaussian distribution with zero mean and standard deviation σ_v known by the sensor.
- Then, assuming that R and Θ_{noise} are independent,

$$\begin{bmatrix} \hat{\xi}_0^+ \\ \hat{\eta}_0^+ \end{bmatrix} = \mathbb{E} \begin{bmatrix} \xi_0 \\ \eta_0 \end{bmatrix} = \mathbb{E}[R] \mathbb{E} \begin{bmatrix} \cos(\hat{\Theta} - \Theta_{\text{noise}}) \\ \sin(\hat{\Theta} - \Theta_{\text{noise}}) \end{bmatrix} + \begin{bmatrix} \xi_{\text{uav}} \\ \eta_{\text{uav}} \end{bmatrix}.$$

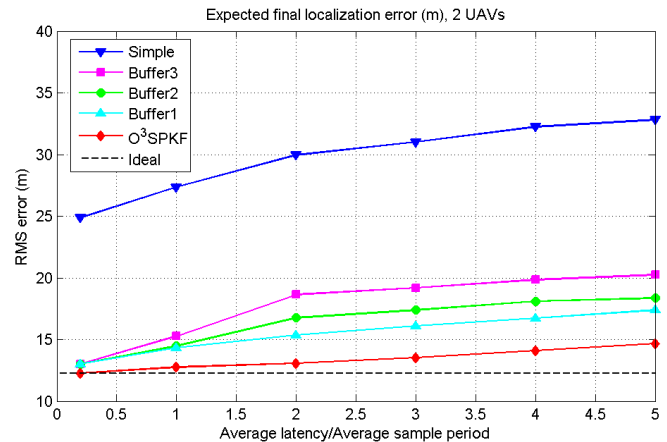
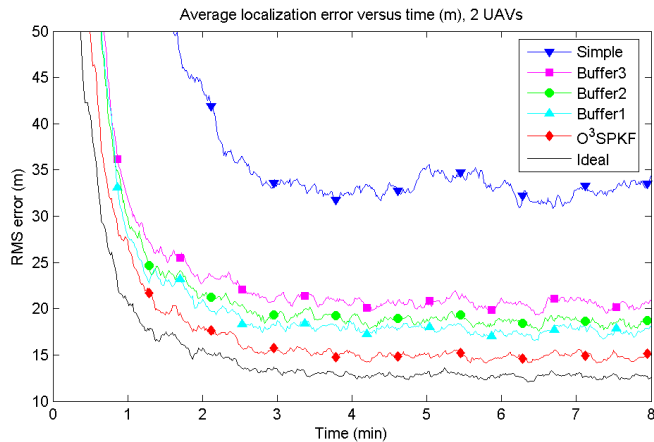
- Without loss of generality, we can assume that the sensor reading $\hat{\Theta} = 0$, and then rotate the final result by the true sensor reading to compensate. For the above assumptions, the final answer is:

$$\begin{bmatrix} \hat{\xi}_0^+ \\ \hat{\eta}_0^+ \end{bmatrix} = \frac{r_0 \exp(-\sigma_v^2/2)}{2} \begin{bmatrix} \cos \hat{\Theta} \\ \sin \hat{\Theta} \end{bmatrix} + \begin{bmatrix} \xi_{\text{uav}} \\ \eta_{\text{uav}} \end{bmatrix}.$$

- Using similar reasoning, the covariance matrix (for these two states) may be found to be:

$$\Sigma_{\tilde{x},0}^+ = \frac{r_0^2}{24} \begin{bmatrix} \cos(\hat{\Theta}) & -\sin(\hat{\Theta}) \\ \sin(\hat{\Theta}) & \cos(\hat{\Theta}) \end{bmatrix} \begin{bmatrix} 4 + 4e^{-2\sigma_v^2} - 6e^{-\sigma_v^2} & 0 \\ 0 & 1 - e^{-2\sigma_v^2} \end{bmatrix} \times \begin{bmatrix} \cos(\hat{\Theta}) & \sin(\hat{\Theta}) \\ -\sin(\hat{\Theta}) & \cos(\hat{\Theta}) \end{bmatrix}.$$

- Furthermore, measurements from cooperating UAVs arrived to the fusion process out-of-sequence due to communication latencies.
- Developed “out-of-order SPKF” (O^3 SPKF) to handle this issue.



- Related ongoing challenges:
 - Knowing UAV (self) position in GPS deprived scenarios.
 - Target modeling with constraints (e.g., railroad problem), and robust estimation for same.

SOC estimation for battery cells

- State: SOC, polarization voltages, hysteresis voltages⁴
 - Observations: Current, temperature, and voltage under load.
 - Issues: Lack of available simple battery model, nonlinear dynamics and measurement.
- Created empirical model of battery cells based on observed phenomena.

⁴ Plett, G., “Sigma-Point Kalman Filters for Battery Management Systems of LiPB-Based HEV Battery Packs—Part 1: Introduction and State Estimation,” *Journal of Power Sources*, Vol. 161, No. 2, October 2006, pp. 1356–68.

- State-of-charge z_k is captured by one state of the model.

$$z_k = z_{k-1} - \Delta T i_{k-1} / C,$$

where current is i_{k-1} , ΔT represents the inter-sample period (in seconds), and C represents the cell capacity (in ampere-seconds).

- The time-constants of the cell voltage response are captured by several filter states. If we let there be n_f time constants, then

$$f_k = A_f f_{k-1} + B_f i_{k-1}.$$

- The hysteresis level is captured by a single state

$$h_k = \exp\left(-\left|\frac{\eta_i i_{k-1} \gamma \Delta T}{C}\right|\right) h_{k-1} + \left(1 - \exp\left(-\left|\frac{\eta_i i_{k-1} \gamma \Delta T}{C}\right|\right)\right) \text{sgn}(i_{k-1}),$$

where γ is the hysteresis rate constant.

- The overall model state is

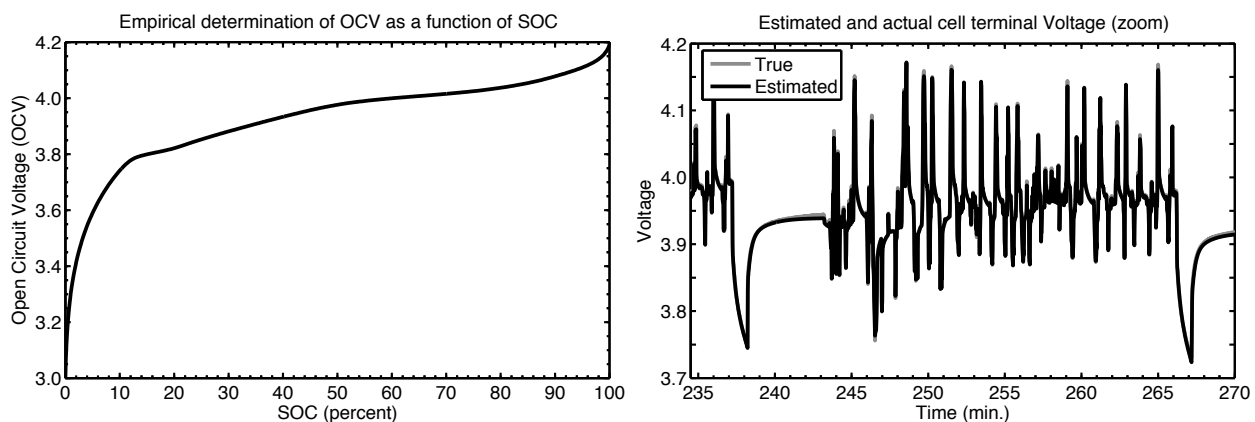
$$x_k = \left[f_k^T, h_k, z_k \right]^T.$$

- The output equation predicts cell voltage

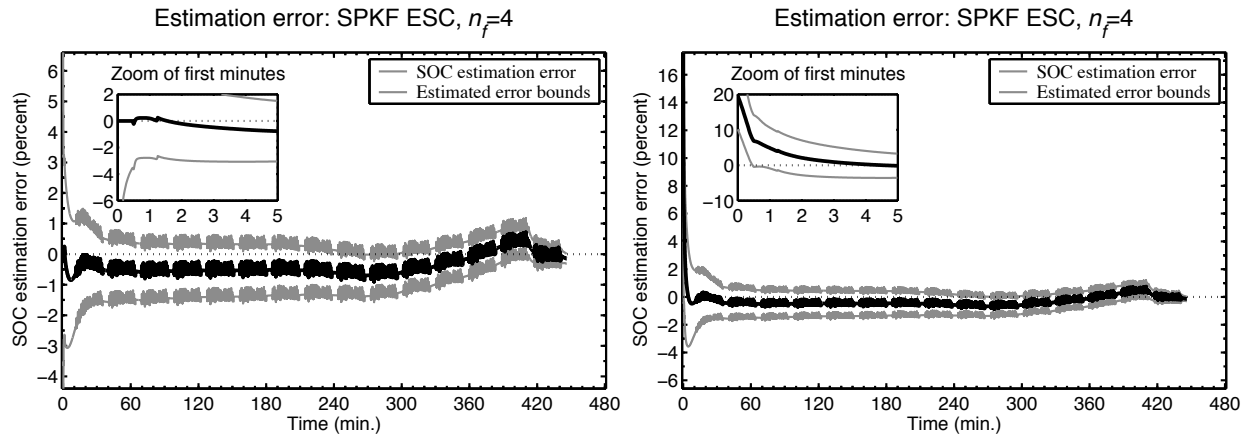
$$y_k = \text{OCV}(z_k) + G f_k - R i_k + M h_k,$$

where G blends the time-constant states together in the output, R is the cell resistance, and M is the maximum hysteresis level.

- Sample results from system identification are:



- Sample results from SPKF state estimation are:



- Comparing EKF to SPKF for this application

	Correctly Initialized			Incorrectly Initialized	
	RMS error	Maximum error	Bounds error	RMS error	Bounds error
EKF	0.64%	1.10%	25.27%	0.75%	6.14%
SPKF	0.49%	0.90%	4.89%	0.69%	2.14%
Improvement	23%	18%	81%	8%	65%

- Related ongoing challenges include

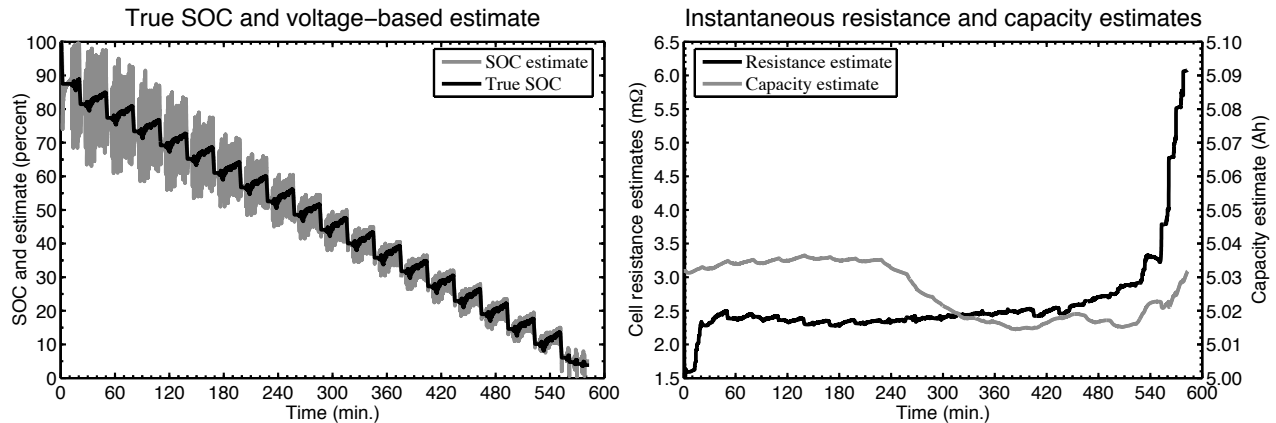
- Empirical model doesn't always work. Need physics-based model.
- Which brings up issues of efficiency of estimation, and needing to define reduced-order (surrogate) models since the physics-based models tend to have hundreds of states.

SOH estimation for battery cells

- State: Resistance and capacity of battery cells⁵

⁵ Plett, G., "Sigma-Point Kalman Filters for Battery Management Systems of LiPB-Based HEV Battery Packs—Part 2: Simultaneous State and Parameter Estimation," Journal of Power Sources, Vol. 161, No. 2, October 2006, pp. 1369–84.

- Observations: Current, temperature, and voltage under load.
- Issues: Parameter estimation; not state estimation
- Dual and joint estimation employed to estimate parameters of cells.
- Surrogate SOC estimate needed to keep states honest. Used “Tino” estimate: $\hat{z} = OCV^{-1}(y + iR)$.



- Related ongoing challenges include
 - Compute available power, energy while minimizing cell damage.
 - Cell capacity is nearly unobservable: Need to estimate robustly.

Localize myself (navigation)

- State: Position, orientation^a
 - Observations: Accelerations, rotations, GPS fixes
 - Issues: Correct for drift of IMU using GPS
- Particular implementation in mind is for a hovering UAV platform.



^a From, Matthew Johnson, *Practical Implementation of a Low Cost Solid-State Gyro-Less Attitude Determination System*, MSEE project, University of Colorado at Colorado Springs, 2009.

Summary

- Kalman filtering is an important tool that finds application in many very different scenarios.
- Lots of applications still waiting to be explored.
- My experience is that every application requires some re-defining of the KF—there's still theory to be developed in defining how to apply KF to different scenarios.
- I look forward to seeing what you do with it!