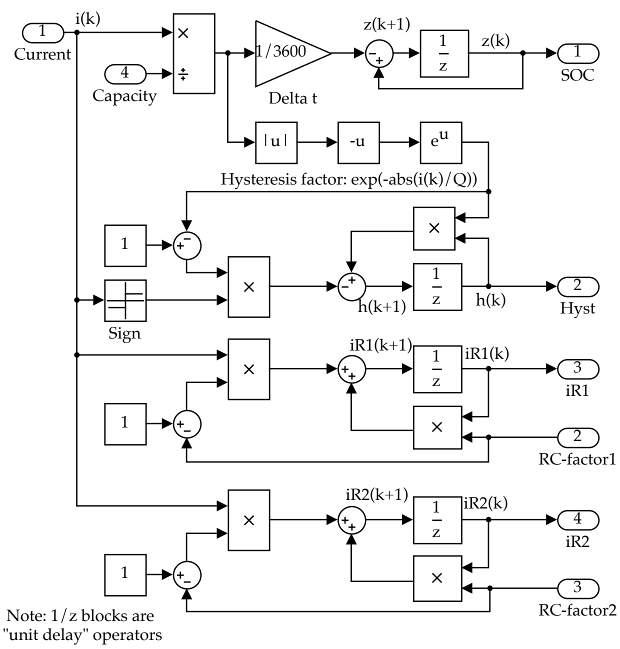
Errata

- Page 55, Fig. 2.23. The summation block that produces $h[k+1]$ has an incorrect sign. The correct figure is (thanks to Ørjan Gjengedal for pointing this out):



- Page 125: In Table 3.2, A_1 should be A_0; A_2 should be A_1 ... and so forth up to A_{15} should be A_{14} (the subscript indices are one too large). Note that the equation for computing OCP often has a singularity right at $\theta = 0.5$, so this input should either be avoided or the equation should be evaluated in the limit at this point.

- Page 205: Approximately middle of page:

$$\frac{\widetilde{C}_{s,\text{avg}}(s)}{J(s)} = \frac{-\text{res}_0}{s} = \frac{-3/R_s}{s}$$

should be

$$\frac{\widetilde{C}_{s,\text{avg}}(s)}{J(s)} = \frac{\text{res}_0}{s} = \frac{-3/R_s}{s}.$$

- Page 205: Immediately after prior correction, $\text{res}_0 = 3 \times 10^5$ should be $\text{res}_0 = -3 \times 10^5$.

- Page 221: Near the middle of the page, the boundary conditions for ϕ_e were miscopied from Sect. 4.11.2 and should be multiplied by negative one. They should be:

$$
\begin{aligned}
-\kappa_{\text{eff}}\frac{\partial\phi_e}{\partial z} - \kappa_{D,\text{eff}}\frac{\partial \ln c_e}{\partial z}\bigg|_{z=0} &= 0 \\
-\frac{\kappa_{\text{eff}}}{L}\frac{\partial\phi_e}{\partial z} - \frac{\kappa_{D,\text{eff}}}{L}\frac{\partial \ln c_e}{\partial z}\bigg|_{z=1} &= \frac{i_{\text{app}}}{A}.
\end{aligned}
$$

- Page 221: Similarly, the simplification immediately following should be:

$$
\frac{\partial\phi_e}{\partial z}\bigg|_{z=0} = 0 \quad \text{and} \quad -\frac{\kappa_{\text{eff}}}{L}\frac{\partial\phi_e}{\partial z}\bigg|_{z=1} = \frac{i_{\text{app}}}{A}.
$$

  These boundary-condition errors do not propagate, and the correct boundary conditions for $\phi_{s\text{-}e}$ are given on the top of page 222.

- Page 274: Margin note 5. This result is actually from Bird (19.3–3) in combination with entry (Q) of Table (17.8–1).

Appendix: Ion Flux Equation

The proof of the ion-flux equation on pp. 102ff is awkward and indirect (although correct). Many thanks to Dr. John Milios from Sendyne Corporation for sending an improved proof upon which the following is based.

Recall Eq. (3.35), which states

$$c\nabla\mu_e = K_{0+}(v_0 - v_+) + K_{0-}(v_0 - v_-),$$

which can be rearranged as

$$\begin{aligned} -K_{0+}v_+ - K_{0-}v_- &= c\nabla\mu_e - v_0(K_{0+} + K_{0-}) \\ -\frac{K_{0+}}{c_+}\mathbf{N}_+ - \frac{K_{0-}}{c_-}\mathbf{N}_- &= c\nabla\mu_e - v_0(K_{0+} + K_{0-}), \end{aligned}$$

where we remember that flux equals concentration multiplied by velocity.

Recall also Eq. (3.24), which states

$$\begin{aligned} \mathbf{i} &= F\sum_i z_i\mathbf{N}_i \\ \mathbf{i}/F &= z_+\mathbf{N}_+ + z_-\mathbf{N}_-. \end{aligned}$$

We can combine both of these into a single matrix equation

$$\begin{bmatrix} -\frac{K_{0+}}{c_+} & -\frac{K_{0-}}{c_-} \\ z_+ & z_- \end{bmatrix} \begin{bmatrix} \mathbf{N}_+ \\ \mathbf{N}_- \end{bmatrix} = \begin{bmatrix} c\nabla\mu_e - v_0(K_{0+} + K_{0-}) \\ \mathbf{i}/F \end{bmatrix}.$$

Solving for the fluxes gives

$$\begin{aligned} \begin{bmatrix} \mathbf{N}_+ \\ \mathbf{N}_- \end{bmatrix} &= \begin{bmatrix} -\frac{K_{0+}}{c_+} & -\frac{K_{0-}}{c_-} \\ z_+ & z_- \end{bmatrix}^{-1} \begin{bmatrix} c\nabla\mu_e - v_0(K_{0+} + K_{0-}) \\ \mathbf{i}/F \end{bmatrix} \\ &= \frac{1}{-\frac{K_{0+}z_-}{c_+} + \frac{K_{0-}z_+}{c_-}} \begin{bmatrix} z_- & \frac{K_{0-}}{c_-} \\ -z_+ & -\frac{K_{0+}}{c_+} \end{bmatrix} \begin{bmatrix} c\nabla\mu_e - v_0(K_{0+} + K_{0-}) \\ \mathbf{i}/F \end{bmatrix} \\ &= \frac{c_-}{z_+}\frac{1}{K_{0+} + K_{0-}} \begin{bmatrix} z_- & \frac{K_{0-}}{c_-} \\ -z_+ & -\frac{K_{0+}}{c_+} \end{bmatrix} \begin{bmatrix} c\nabla\mu_e - v_0(K_{0+} + K_{0-}) \\ \mathbf{i}/F \end{bmatrix}, \end{aligned}$$

remembering that $z_+c_+ = -z_-c_-$. Simplifying, we have

$$\begin{bmatrix} \mathbf{N}_+ \\ \mathbf{N}_- \end{bmatrix} = \frac{1}{K_{0+} + K_{0-}} \begin{bmatrix} -c_+ & \frac{K_{0-}}{z_+} \\ -c_- & \frac{K_{0+}}{z_-} \end{bmatrix} \begin{bmatrix} c\nabla\mu_e - v_0(K_{0+} + K_{0-}) \\ \mathbf{i}/F \end{bmatrix}.$$

Solving for $\mathbf{N}_+$ in particular,

$$\mathbf{N}_+ = \frac{-c_+c}{K_{0+} + K_{0-}}\nabla\mu_e + c_+v_0 + \underbrace{\frac{K_{0-}}{K_{0+} + K_{0-}}}_{t_+^0}\frac{\mathbf{i}}{z_+F}.$$

This is almost in the desired form. For the first term, we recall Eq. (3.36), which states

$$\mathscr{D} = \frac{\nu R T c_0 c}{c_T(K_{0+} + K_{0-})}.$$

Substituting and recognizing that $c_+/c = \nu_+$ gives

$$\mathbf{N}_+ = \frac{-\nu_+ \mathscr{D}}{\nu R T}\frac{c_T}{c_0} c \nabla \mu_e + \frac{\mathbf{i} t_+^0}{z_+ F} + c_+ v_0,$$

which is what we set out to prove. Similar steps are used to simplify $\mathbf{N}_-$ from the matrix equation to give

$$\mathbf{N}_- = \frac{-\nu_- \mathscr{D}}{\nu R T}\frac{c_T}{c_0} c \nabla \mu_e + \frac{\mathbf{i} t_-^0}{z_- F} + c_- v_0.$$